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ABSTRACT

The educational system is characterized by a vector matrix differential equation, and it is shown that the concepts of modern control theory such as observability, controllability, and optimal control can give deeper insight into the behavior of the educational system. The identification of the system dynamics and the definition of a realistic objective function are two major stumbling blocks in the way of progress. (Author/IRT)

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Control Concepts in Educational Planning

By

N. D. FRANCIS

EA 008 022

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JANUARY 1972

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N. D. Francis

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CONTROL CONCEPTS IN EDUCATIONAL PLANNING

ABSTRACT

The educational system is characterised by the Vector metrix differential equation,

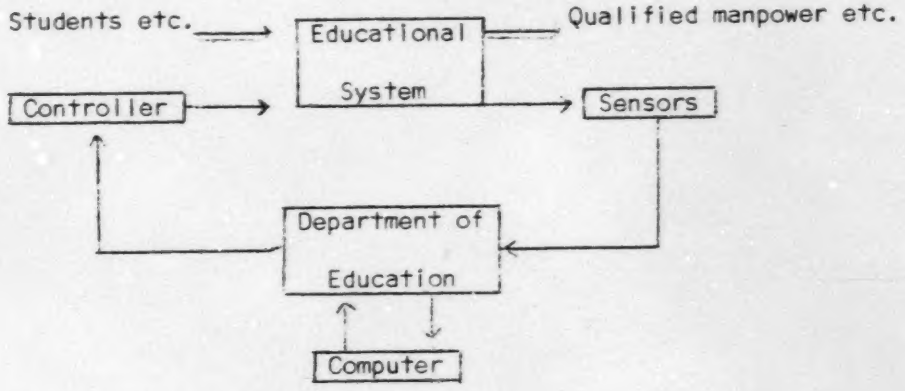
$$\frac{dx}{dt} = Ax + Dm$$

and it is shown that the concepts of modern control theory such as observability, controllability, optimal control etc. can give deeper insight into the bahaviour of the educational system. The identification of the system dynamics and the definition of a realistic objective function are two major stumbling blocks in the way of progress.

A = n X n matrix ; n >> n

1. Introduction

A system[4,5] can be defined as a collection of objects inter-related to each other and produce various outputs for different inputs. A set of mathematical equations capable of predicting the behaviour of the system under varying conditions is essential in order to run the system according to our liking. For an educational system, flow of students, teachers, equipments etc. form the input variables while the output consists of qualifications, emigration, death etc. A modern high speed digital computer plays a dominant role in the design and analysis of an Educational system which involves a large amount of numerical data.



It is important to note that the Department of Education forms part of the closed loop control system which governs the educational system. There are various types of systems (linear, nonlinear etc.) and these are identified by analysing the structure, the type of nonlinearites and the numerical values of associated parameters. The methods available for the identification of multivariable systems have met with varying degrees of success, when applied to scientific

systems. The extension of these methods to socio-economic systems, especially to those systems which involve human beings, present a lot of difficulties and perhaps this is the major reason why the educationalists are pessimistic about the applications of Systems theory into educational planning.

However, the development of a unified systems theory for solving practical problems is very promising. It is a known fact that the human system is the most complex system in the whole world and the interpretation of this system on the basis of the present under-developed system's theory can not occur overnight. This is going to be an evolutionary process and the present systems theory which is proved to be a success in the field of engineering and science can be extended as and when we know more about our human system. The author believes that reality can be achieved only through unreality and efforts should be made in extending the concepts of systems theory into educational planning, in spite of the present difficulties.

Almost all systems we see around us are subjected to various types of disturbances and there is a tendency to drift away from the desired operating point. Therefore, the given system can give a satisfactory performance only if there is a built in mechanism to apply a corrective action on the basis of the difference between the actual output and the desired output. This is the principle behind the design of feedback control systems. In the following sections, we will discuss the applications of modern control theory in the design and analysis of educational systems.

2. State-space methods for Systems analysis:-

The state of dynamic system is defined as the quantitative information which is the least amount of data which one requires to know about the system in order to predict its future behaviour. The behaviour of the linear dynamic system can be described by the solution of a set of linear first order differential equations whose properties can be studied by using the theory of matrices. These first order differential equations describe the state of the system and thus give an account of the energy stored in a set of independent energy storage cells. Consequently the total energy stored in the system can be determined by knowing the state variables and to some extent the state of the system summarises the contributions of the past that are important for the future.

All the state variables of a system need not be accessible for measurement and observation and here the problem of estimation of the state vector X using the output vector Y arises. Obviously, much more efficiency and accuracy can be obtained if it is possible to choose measurable quantities as state variables. Fortunately, state-space representation of a system is not unique and this principle can be used to select measurable quantities as state variables, in many cases.

The state-space representation of a n^{th} order system defined by

$$\frac{d^n x(t)}{dt^n} + a_1 \frac{d^{n-1} x(t)}{dt^{n-1}} + \ldots + a_n x(t) = m_n(t)$$

is given by

$$\frac{d\,X(t)}{dt} = A\,X(t) + Ib$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ \cdot & & & & & & \\ \cdot & & & & & & \\ \cdot & & & & & & \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & & & & & -a_1 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ m_n \end{bmatrix} \quad ; \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$$

and $x = x_1$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\cdot$

$\cdot$

$\cdot$

$\dot{x}_{n-1} = x_n$

$\dot{x}_n = -a_1 x_n - a_2 x_{n-1} \cdots - a_n x_1 + m_n$

2.1 Nonuniqueness of State-space representation

Consider the system described by the equations

$$\frac{d^3z(t)}{dt^3} + \frac{2\,d^2z(t)}{dt^2} + \frac{dz(t)}{dt} = \frac{d\,m_4(t)}{dt} + m_4(t) \qquad \text{(a)}$$

$$\frac{d^2x(t)}{dt^2} + \frac{d\,x(t)}{dt} = Z(t) \qquad \text{(b)}$$

where x, $\dot{x}$, z, $\dot{z}$ are measurable quantities. combining (a) and (b), we have

$$\frac{d\,x}{dt} = A\,X + I\,b \quad \text{----} \qquad \text{(c)}$$

where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$; $x = x_1$; $\dot{x}_1 = x_2$; $\dot{x}_2 = x_3$; $\dot{x}_3 = x_4$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ m_4 \end{bmatrix}$$

The state variables x_3 and x_4 are not accessible for measurement in the above state-space representation of the system.

We will now try to find an alternate representation in which all the state variables are measurable.

Let $x_1 = y_1 = x$

$x_2 = \dot{x}_1 = y_2$

$x_3 = \dot{x}_2 = y_3 - y_2$ where $y_3 = z$

$x_4 = \dot{x}_3 = y_2 - y_3 + y_4$ where $y_4 = \dot{y}_3$

Substituting $Y = MX$ in (c) one gets

$$\frac{dy}{dt} + \bar{A}y + Mb$$

where

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$$Mb = \begin{bmatrix} 0 \\ 0 \\ 0 \\ m_4 \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

All elements of the state vector Y are accessible for measurement and thus we have established the nonuniqueness of state-space representation and the importance of selecting measurable quantities as state-variables.

3. Linear Systems:-

Let us consider an educational system whose state at any
can be described by a vector $\bar{V}(t)$ belonging to a finite dime
Euc[illegible] space V_{n+s} where $s \leq n$; $\bar{V} = [(\bar{m})', (\bar{X})']'$
$\bar{m}$ = s x 1 input vector; $\bar{X}$ = n x 1 state vector.
The compone[illegible]s of m represent the input to the educational sy
such as students, teachers, equipments etc. while the element
X might represent the stock of students in different grades,
of qualified people, emigration, death, social status, natura
environments etc. Some of these variables can be measured di
(number of students, number of qualified people etc.) while s
are not accessible for direct measurements (e.g. attainments,
environments, family background) and are to be estimated usin
available information.

We assume that any future state of the educational syste
only upon its present state and not upon its past history (we
claim this assumption to be perfectly true). This assumptio
to the mathematical formulation

$$\frac{d\bar{X}}{dt} = f[\bar{v}, t] \quad \ldots\ldots (1)$$

The function f is in general nonlinear and time varying. Th
identification of the educational system involves the determ
of the structure of f (ordinary differential equations, part
differential equations, differential - difference equations
and the numerical evaluation of its parameters. Even though
inadequate at the moment, attempt can be made to determine t
simulation studies, theoretical considerations or by the act

analysis of performance data. The study of the nonlinear function f can be simplified by splitting the original model into its components - steady state model and dynamic model.

If the variation of f is very small compared to time and the system variables over the time interval under consideration, then (1) can be approximated to

$$\frac{d\bar{X}(t)}{dt} = f\left[\bar{v}(t)\right]$$

$$\bar{v}(0) = v_0$$

It can be shown that

$$\frac{dX}{dt} = f\left[v^e\right] + \left[Dm + Ax\right] \quad \cdots\cdots (2)$$

= steady state model + dynamic model

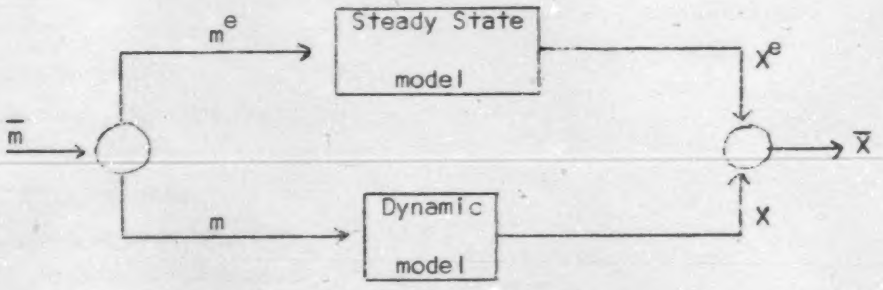
where $\bar{x} = X^e + X$; $\bar{v} = v^e + v$; $\bar{m} = m^e + m$; v,x,m, are variations about the equilibrium points v^e, X^e, and m^e respectively, and

D = n x s driving matrix

A = n x n System "

m = s x 1 Input vector (control vector)

X = n x 1 State vector.



At the equilibrium point $f\left[v^e\right] = 0$ and therefore (2) becomes

$$\frac{dX}{dt} = AX + Dm \quad \cdots\cdots (3)$$

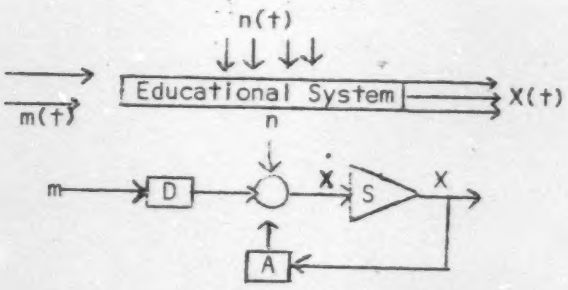
which is the linear dynamic model of the educational system.

When the f is a function of time, the model takes the form

$$\frac{dX(t)}{dt} = A(t)\,X(t) + D(t)\,m(t) \qquad (4)$$

In actual practice systems are not free from noise (disturbances due to student unrest, social, economic and political changes etc) and thus (4) can be modified to include $n(t)$ which is assumed to be a $n \times 1$ independent randum noise vector with zero mean (the method given here cannot be applied for any noise $n(t)$

$$\frac{d\,X(t)}{dt} = A(t)\,X(t) + D(t)m(t) + n(t) \qquad (5)$$



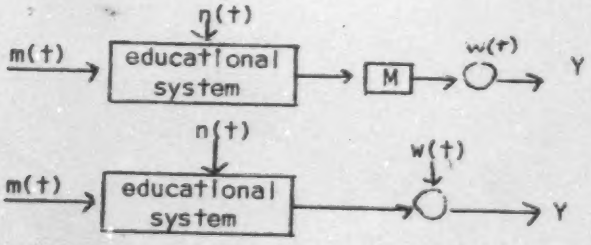
We have already mentioned that all state variables need not be accessible for measurement and in some cases we can derive a relationship of the form

$$Y = M\,X + w \qquad (6)$$

when $M = p \times n$ measurement matrix

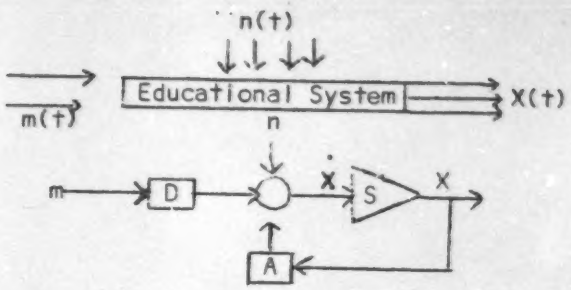
$Y = p \times 1$ output vector; $p \leq n$

w = independent randum measurement noise vector with zero mean. w includes errors due to uncertainty, different accounting methods etc.



Obviously, X and Y are identical only if $p = n$, $w = o$ and $M = I$ = identity matrix.

$$\frac{\quad}{dt} = A(t)\,X(t) + D(t)m(t) + n(t) \qquad (5)$$



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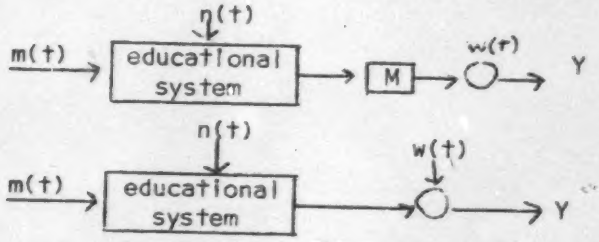


Obviously, X and Y are identical only if p= n, w = o and M = I = identity matrix.

4. Linear Systems Analysis

The solution of the linear dynamic system

$$\frac{dX(t)}{dt} = A(t)\,X(t) + D(t)\,m(t) + n(t)$$

is given by

$$X(t) = F(t,t_o)\,X(t_o) + \left[\int_{t_o}^{t} F(t,t_1)\,D(t_1)\,m(t_1)\,dt_1 + \int_{t_o}^{t} F(t,t_1)\,n(t_1)\,dt_1\right]$$

= Steady state solution + Transient solution.

When the control vector is piecewise constant

$$m(t_1) = m(t_k) \text{ for } t_k \leq t_1 \leq t_{k+1}$$

and this leads to

$$X(\overline{k+1}\,T) = F(kT)\,X(kT) + G(kT)m(kT) + U(kT)$$

$$X(\overline{k+1}) = F(k)\,X(k) + G(k)\,m(k) + U(k) \qquad (8)$$

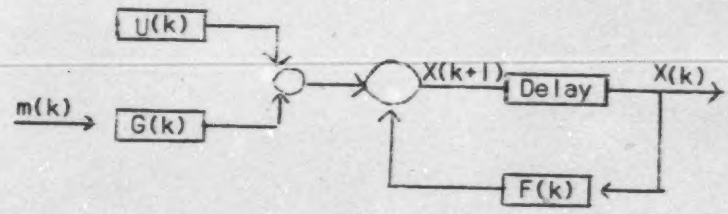
$t_{k+1} - t_k = T$ = sampling period

$$F(k) = F(t_{k+1}, t_k) = F(\overline{k+1}\,T, kT)$$

$$G(k) = \int_{kT}^{k+1\,T} F(\overline{k+1}\,T, t_1)\,D(t_1)\,dt_1$$

$$U(k) = \int_{kT}^{\overline{k+1}\,T} F(\overline{k+1}\,T, t_1)\,n(t_1)\,dt_1$$

ie a recursive relation to compute the state at time t_{k+1} from the state at t_k and the forcing functions (inputs)



4.1 Stationary System:-

When A and D are time invariant matrices

$$F(k) = F(t) = e^{AT} = \sum_{j=0}^{\infty} \frac{A^j T^j}{j!} = e^{T^*}$$

$$= I + T^* + \frac{T^*}{2} + \frac{(T^*)}{1!} + \frac{T^*}{3}\frac{(T^{*2})}{2!} + \ldots + \frac{T^*}{n}\frac{(T^{*n-1})}{(n-1)!} \quad (9)$$

$$= p_1 e^{\lambda_1 T} z_1^o + p_2 e^{\lambda_2 T} z_2^o + \ldots p_n e^{\lambda_n T} z_n^o \quad (10)$$

Where $P = [p_1\ p_2 \ldots p_n]$ = eigenvector matrix of A

$X = PZ = P[z_1 z_2 \ldots z_n]^1$

$z_i(t_o) = z_i^o$

eigenvalues λ_i are assumed to be distinct.

The equations (9) and (10) give the same transient solution; but characteristic response of different modes are not seperated in (9) as in (10) and thus (9) fails to give insight into the system behaviour and is valid only when T is small. This solution (10) is a linear combination of the n distinct modes $p_i e^{\lambda_i T}$ of the educational system and attempt can be made to suppress undesirable modes by selecting suitable initial conditions $Z^o = P^{-1} X_o$. Educational decision makers can take advantage of this situation while designing new systems or introducing modifications to the existing systems. The system matrix A contains complete information on the system dynamic modes and the analysis of A determines the dynamic characteristic and stability properties of the system.

4.3 Simplification of systems model:-

We have already mentioned that a class of multivariable systems can be approximated by (3). Even, very simple and innocent looking physical systems generally require a large number of simultaneous differential equations (say 10000) for their representation. It is impossible to work with such high order systems and therefore the reduction of order of large systems is an essential feature of mathematical modelling.

A model of the form

$$\frac{dX^c}{dt} = A^c x^c + D^c m^c$$

can be simplified to the form (3) in some cases, where

$A^c = n^c \times n^c$ matrix ; $n^c \gg n$
$D^c = n^c \times S^c$ " ; $S^c \leq n^c$
$X^c + n^c \times 1$ vector ; $S^c > S$
$M^c = S^c \times 1$ "

The educational systems involve a large number of equations and the structure of the system (i.e. properties of A) decides whether it can be simplified preserving the characteristics of the original system.

4.4 Sensitivity Studies:-

The eigenvalue trajectories throw light on the behaviour of the system due to variations in the system parameters. The stability and speed of response of the educational system can be increased by moving the eigenvalues of the System matrix A in the negative direction, preferably at a uniform rate. We are particularly interested in the movements of the most + ve eigenvalues which cause the slower response of the system. Some eigenvalues are more sensitive to variation of certain parameters than others and by knowing this, we can take special precautions while determining these parameters and also in the measurement and control of the variables influenced by them. This approach helps study the behaviour of the system and the educational policy makers can avoid

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decisions that bring about chaos in the educational system and the society in general. It can be shown[6] that

$$\delta x = \left\{ \text{trace} \left[\hat{Q}(\lambda_r) \right] \right\}^{-1} \left[\hat{Q}(\lambda_r) \right] * \left[(SA) \right]$$

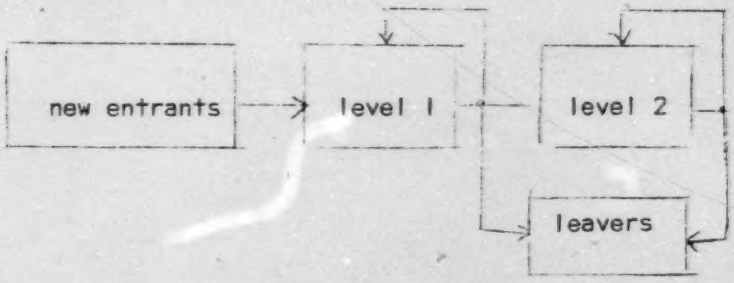
where * denotes the inner product of mitrices and Q is the adjoint matrix given by

$$\hat{\Phi}(\lambda) = \left| (\lambda_r I - A) \right| \left[\lambda_r I - A \right]^{-1}$$

λ_r = distinct eigenvalues

5. A Simple Educational System

Let us consider an educational system made up of two levels of education, shown below



The algebraic representation of this block-diagram is given in Table 1.

Table 1

		levels		Leavers	
t + 1 \ t		1	2	1	total at time t
LEVELS	1	e_{11}	e_{12}	ℓ_{11}	S_1, t
	2	e_{21}	e_{22}	ℓ_{21}	S_2, t
New Entrants		N_1			
Total at time (t + 1)		$S_1, t+1$	$S_2, t+1$	L_1, t	

Also we have

$$G(K) = G(T) = \int_0^T e^{A(T-t_1)} D\, dt_1$$

$$= \left[\sum_{j=0}^{\infty} \frac{A^j T^{j+1}}{(j+1)!}\right] D$$

$$= \left[(e^{AT} - I)A^{-1}\right] D \qquad (10a)$$

$$U(k) = \int_{KT}^{\overline{k+1}\,T} e^{A(\overline{k+1}\,T - t_1)} n(t_1)\, dt_1$$

4.2 Nonstationary systems:-

Here, the matrices A and D are functions of time and in this case $A(t)$ and $\int_{t_0}^{t} A(t_1)\, dt_1$ do not commute and a solution of the form

$$F(t,t_0) = e^{\int_{t_0}^{t} A(t_1)\, dt_1}$$

can be attempted only if $A(t)$ is a diagonal matrix. The solution of the time varying system is given by the matrizant.

$$F(t,t_0) = \left[I + \int_{t_0}^{t} A(t_1)\, dt_1 + \int_{t_0}^{t} A(t_1)\, dt_1 \int_{t_0}^{t} A(t_2)\, dt_2 + \int_{t_0}^{t} A(t_1) dt_1 \right.$$

$$\left. \int_{t_0}^{t_1} A(t_2)\, dt_2 \int_{t_0}^{t_2} A(t_3)\, dt_3 + \ldots\right]$$

which is an infinite series of successive integrals - highly pleasing to the mathematician but not fit for practical computations. In general, there is no standard method for solving this class of problems and depending upon the special nature of coefficients we will have to apply various approximation techniques in order to extract useful information out of the given system.

$S_{i,t}$ = total number of students in level i at time t

= students in level i at the beginning of the academic year (t-1), t

e_{ij} = number of studetns in level i at time t and in level j at time (t+1). In most cases $e_{ij} = 0$ for $j < i$.

N_1 = number of new entrants into standard one in year (t+1)

$L_{1,t}$ = total number of leavers during the year t. ie. during the academic year (t-1), t.

ℓ_{i1} = number of leavers from level i during t

t = base year

$$e_{11} + e_{12} + \ell_{11} = S_{1,t}$$
$$e_{21} + e_{22} + \ell_{21} = S_{2,t}$$
$$\ell_{11} + \ell_{21} = L_{1,t}$$
$$e_{11} + e_{21} + N_1 = S_{1,\,t+1}$$
$$e_{12} + e_{22} = S_{2,\,t+1}$$

Table 1 is used to derive the coefficient matrix shown in Table 2 and the transpose of this coefficient matrix is used to construct the transition matrix F shown in Table 3.

Table 2

	levels		leavers
	1	2	1
1	$\frac{e_{11}}{S_{1,t}}$	$\frac{e_{12}}{S_{1,t}}$	$\frac{\ell_{11}}{S_{1,t}}$
2	$\frac{e_{21}}{S_{2,t}}$	$\frac{e_{22}}{S_{2,t}}$	$\frac{\ell_{21}}{S_{2,t}}$
leavers	0	0	1

$$e_{21} + e_{22} + \ell_{21} = S_{2,t}$$

$$\ell_{11} + \ell_{21} = L_{1,t}$$

$$e_{11} + e_{21} + N_1 = S_{1,\,t+1}$$

$$e_{12} + e_{22} = S_{2,\,t+1}$$

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	1	2	1
1	$\frac{e_{11}}{S_{1,t}}$	$\frac{e_{12}}{S_{1,t}}$	$\frac{\ell_{11}}{S_{1,t}}$
2	$\frac{e_{21}}{S_{2,t}}$	$\frac{e_{22}}{S_{2,t}}$	$\frac{\ell_{21}}{S_{2,t}}$
leavers	0	0	1

Table 3

$\frac{e_{11}}{S_{1,t}}$	$\frac{e_{21}}{S_{2,t}}$	0
$\frac{e_{12}}{S_{1,t}}$	$\frac{e_{22}}{S_{2,t}}$	0
$\frac{\ell_{11}}{S_{1,t}}$	$\frac{\ell_{21}}{S_{2,t}}$	1

Combining Tables 1 and 3 we get

$$\begin{bmatrix} S_{1,t+1} \\ S_{2,t+1} \\ L_{1,t} \end{bmatrix} = \begin{bmatrix} \frac{e_{11}}{S_{1,t}} & \frac{e_{21}}{S_{2,t}} & 0 \\ \frac{e_{12}}{S_{1,t}} & \frac{e_{22}}{S_{2,t}} & 0 \\ \frac{\ell_{11}}{S_{1,t}} & \frac{\ell_{21}}{S_{2,t}} & 1 \end{bmatrix} \begin{bmatrix} S_{1,t} \\ S_{2,t} \\ 0 \end{bmatrix} + \begin{bmatrix} N_1 \\ 0 \\ 0 \end{bmatrix} \qquad (11)$$

which is in the form of equation (8)

The enrolment at time (t+1) (ie. school year t, (t+1)) and the number of school leavers available for the labour force during (t-1),t can be obtained from Table 1 or equation (11). The inter-educational flows in Table 1 show the progress of students within the educational system during a one year period. The school leavers are those who were in the educational system at the start of the given year, but were no longer in the system at the beginning of the next year. The latter group constitutes the entrants to the labour force, emigration, death etc. The total enrolment at time (t+1) and the number of leavers depend upon the school age population (ie. births - demographic factor).

The enrolment in level I at time (t+I) depends upon both new entrants and repeaters in level I and this will have an effect on the enrolment in level 2 at time (t+2) and so on. This chain reaction continues and in a real system the effect of this reaction will be felt only during the course of a few years.

If we use the above transition matrix as a basis for educational planning, it is clear that the future educational enrolments and outputs will be affected by the enrolments in each level at the end of the base year and also the new entrants during the plan period. The new entrants include births, immigrants, foreign students and also those who re-enter the system from the labour force.

Note that the model represented by equation (II) is of the form

$$X(k+1) = \Gamma(k)\, X(k) + G(k)\, m(k)$$

It is possible to convert this into the general form

$$\frac{dx}{dt} = Ax + Dm \quad \ldots\ldots\ldots \qquad (12)$$

We know that

$A = PJP^{-1}$; J = Jordan canonical form, P = matrix of eigenvectors

For a time - invarient system

$$F = e^{A} = P\, e^{J}\, P^{-1}$$

and $\log F = \log (e^{A}) = P \log (e^{J}) P^{-1}$

$$= PJP^{-1} = A$$

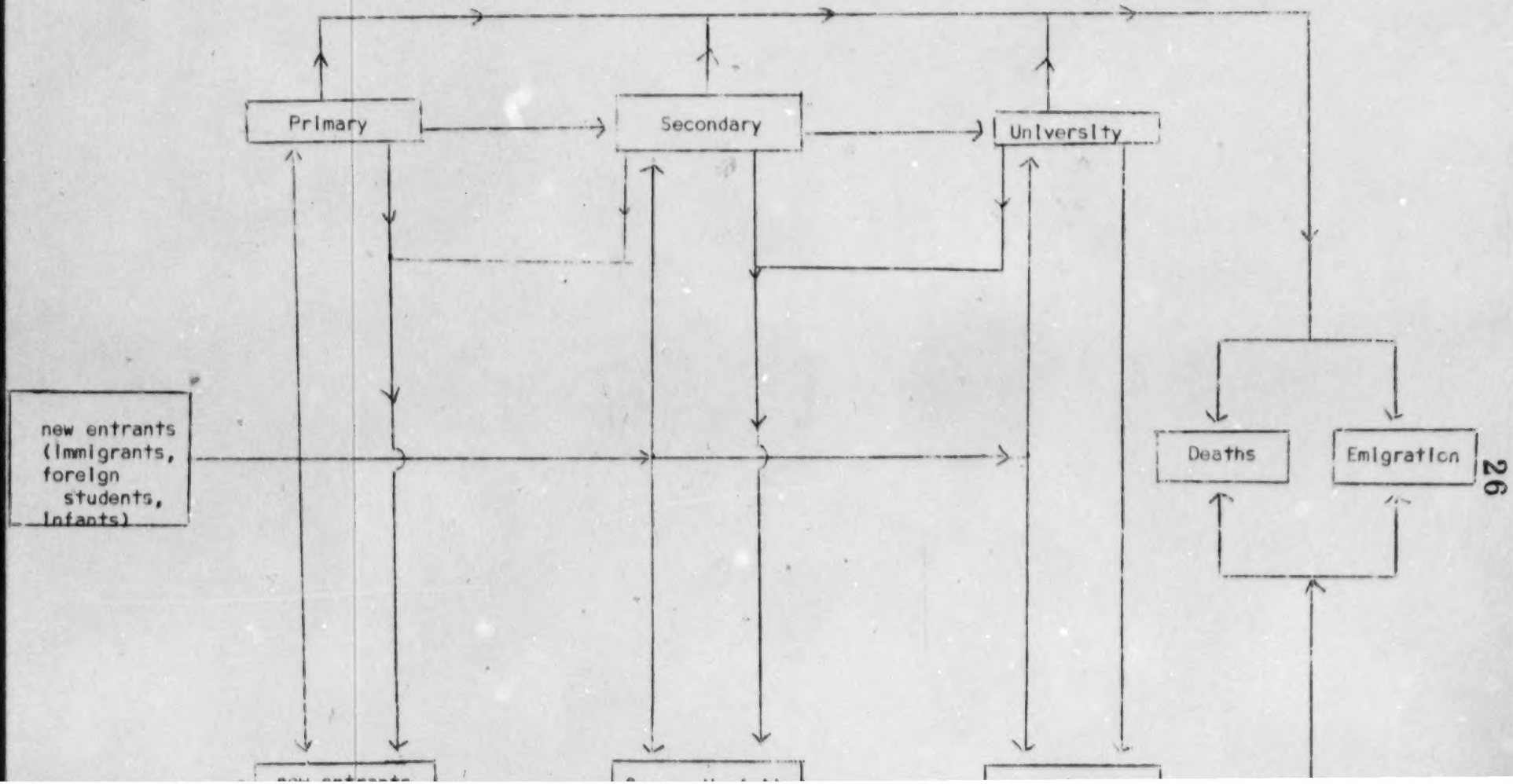
Thus A can be derived from F and the elements of D in (12) can be determined from (10a)

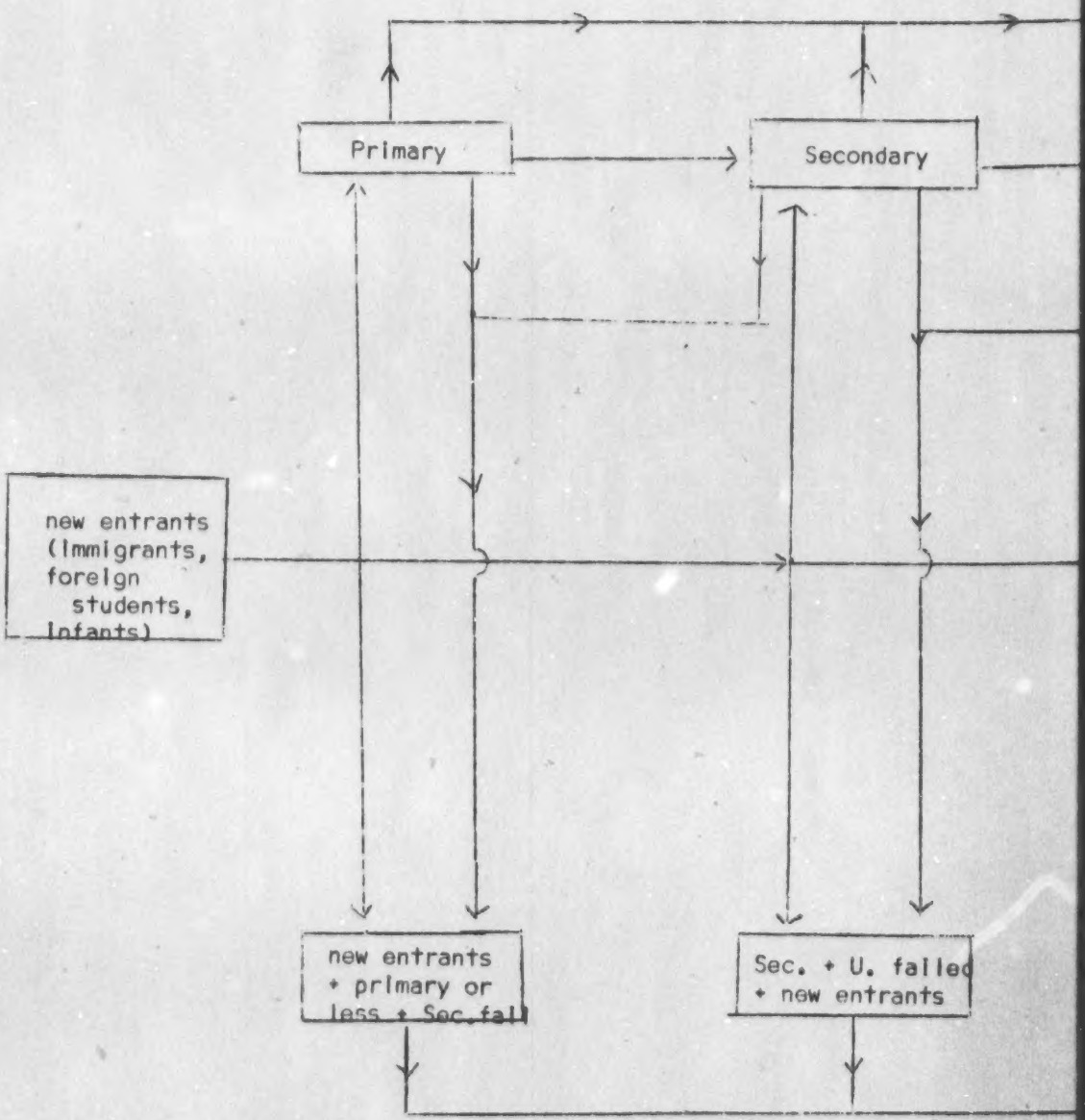
For example, when $F = \begin{bmatrix} .6005 & .2326 \\ -.4652 & -.0973 \end{bmatrix}$

We have, $F = \begin{bmatrix} 2 & 2 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} .3679 & 0 \\ 0 & .1353 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

and $A = \text{Log } F = \begin{bmatrix} 2 & 2 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

The following block-diagram shows an aggregate model of a national educational system.


Primary
Secondary
University
new entrants (immigrants, foreign students, infants)
Deaths
Emigration


Primary
Secondary
new entrants (immigrants, foreign students, infants)
new entrants + primary or less + Sec. fail
Sec. + U. failed + new entrants

A word of caution is in order at this stage. The elements of the transition matrix (see Table 3) were derived from the promotions, detentions etc. and these are assumed to be simple constants. In fact, these elements are functions of many complex variables such as:-

a) Physically and psychologically comfortable home for the students
b) Attitude of the society towards educated persons
c) Resources for education
d) Quality of teachers
e) Laboratories
f) Recreational facilities
g) Libraries
h) Employment opportunities
i) Pressure for learning
j) Health programmes
k) Students' ability and attitudes towards education
l) Structure of the educational system
m) Parents' attitudes towards education
n) Qualities of parents
o) Absence of serious emotional disturbances etc.

Some of these factors are to be included in our model as state variables in order to improve the performance of the model. Unfortunately many of these variables are not accessible for direct measurement and are to be estimated. Resources requirements of the model can also be classified as state variables.

6. Controlability, observability and stability

We have already mentioned that the educational system described in Section 5 can also be characterised by

$$\frac{dX}{dt} = AX + Dm \qquad (13)$$

where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} S_{1,t} \\ S_{2,t} \\ L_{1,t} \end{bmatrix}$

$$A = \begin{bmatrix} -a_{11} & 0 & 0 \\ a_{21} & -a_{22} & 0 \\ a_{31} & -a_{32} & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \quad \begin{bmatrix} N_1 \\ 0 \\ 0 \end{bmatrix}$$

a_{11} = fractions of students who left level 1 at time t

a_{21} = fraction of students who were promoted from level 1 to level 2 at time t.

$a_{32} = a_{22}$ = fraction of students who left level 2 at time t.

$a_{32} + a_{31}$ = fraction of students who left the system at time t.

$a_{11} = a_{21} + a_{31}$

All the state variables associated with an educational system need not be accessible for measurement. Suppose the measurable output Y of a time invariant system is related to the state vector x by

$Y = MX$ and $X = PZ$

M = pXn matrix; $p \leq n$

P = nXn matrix;

Then from (13)

$$\dot{Z} = P^{-1}APZ + P^{-1}Dm \qquad (14)$$

$$Y = MPZ = \bar{M}Z \qquad (15)$$

The concept[1,2,6], of controllability, stability and observability

$$\begin{bmatrix} a_{21} & a_{22} & 0 \\ a_{31} & -a_{32} & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \quad \begin{bmatrix} N_1 \\ 0 \\ 0 \end{bmatrix}$$

a_{11} = fractions of students who left level 1 at time t

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Then from (13)

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$$Y = MPZ = \bar{M}Z \qquad (15)$$

The concept[1,2,6] of controllability, stability and observability

define some fundamental properties of linear systems and play an important role in the practical applications of systems theory.

A state variable is said to be observable if it can be determined from measurable output variables. Obviously, X is always observable if M is a nXn nonsingular matrix; in the case where M = p X n matrix, the state vector is observable only if each column of $\overline{M}$ contains at least one nonzero element. Unobservable systems have dynamic modes of behaviour which can not be identified from measurements of the available outputs.

when, $\overline{M} = \begin{bmatrix} M_{11} & 0 & M_{13} \\ M_{21} & 0 & M_{23} \\ M_{31} & 0 & M_{33} \end{bmatrix}$

the equation (15) gives

$$y_1 = M_{11}Z_1 + M_{13}Z_3$$
$$y_2 = M_{21}Z_1 + M_{23}Z_3$$
$$y_3 = M_{31}Z_1 + M_{33}Z_3$$

The modified state vector Z_2 can not be determined from the above equations and thus the system (13) is not completely observable.

In general, a system is completely observable if, for any time t_o and t_f, every state $X(t_o)$ can be determined from the output $Y(t)$ observed over the interval $t_o \leq t \leq t_f$. observability indicates the ability of each state variable to influence the output of the systems

test for observability[3]

The system is completely observable in the interval $t_o \leq t \leq t_1$ if

$\int_{t_o}^{t_1} F^1(t,t_o)\, M^1(t)\, M(t)\, F(t,t_o)\, dt$ is nonsingular (for timevarying systems)

or

$\left[M^1 \vdots A^1M^1 \vdots (A^1)^2 M^1 \vdots \cdots\cdots \vdots (A^1)^{n-1} M^1 \right]$ is of rank n (for timeinvarient systems)

Needless to say that if possible it is desirable to select measurable quantities as state variables. A system is state controllable if there exists a control vector m(t) which move the system from some initial state $X(t_o)$ to a desired state $X^D(t_1)$ in a finite time interval $(t_1 - t_o)$; $t_o \leq t \leq t_1$

Substituting $J = P^{-1}AP$ = jordan canonical form (eigenvalues of A are assumed to be distinct) in (14)

$$\frac{dz}{dt} = JZ + P^{-1}Dm \qquad (16)$$

The system (16) is controllable only if each vow of the matrix $(P^{-1}D)$ contains at least one nonzero element.

when, $$P^{-1}D = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ 0 & 0 & 0 \\ P_{31} & P_{32} & P_{33} \end{bmatrix} ; \quad J = \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & J_3 \end{bmatrix}$$

the equation (16) gives

$$\dot{Z}_1 = J_1Z + P_{11}m_1 + P_{12}m_2 + P_{13}m_3$$

$$\dot{Z}_2 = J_2Z$$

$$\dot{Z}_3 = J_3Z + P_{31}m_1 + P_{32}m_2 + P_{33}m_3$$

The state variable Z_2 can not be controlled by the input vector m and hence the system (13) is not completely controllable. Controllability implies the ability of some finite input to influence each state variable. Conversely, uncontrollable systems have dynamic modes of behaviour which can not be influenced by the input variable.

Test for controllability[3]:-

A system is completely state controllable in the interval $t_o \leq t \leq t_1$, if

$\int_{t_o}^{t_1} F(t_o,t)\, D(t)\, D^1(t)\, F^1(t_o,t)\, dt$ is non singular (for timevarying systems) 32

or

$[D : AD : A^2D : \text{-----} : A^{n-1}D]$ is of rank n (for time invariant systems)

State controllability does not mean output controllability. It can be shown that if the rank of the matrix M is p, then state controllability ensures output controllability while the reverse is not always true.

The relation (16) clearly isolates those variables that can be influenced by educational policy makers from those which can never be controlled by them and this may indicate the necessity for changing the structure of the educational system.

Even if the system is observable and controllable, it need not be stable. The system undergoes violent oscillations and may cause political, economic and social instability. If all the eigenvalues of A have negative real parts, then the system relansed from a set of initial conditions tends to move towards the equilibrium state as $t \longrightarrow \infty$.

The structure of A and D determine the fate of the system and this information can be used to design a well balanced educational system.

7. OPTIMAL CONTROL

A linear dynamic system is characterised by

$$\frac{dx}{dt} = f\left[x(t), m(t), n(t), t\right]$$

$$m(t) \in \text{—}\tilde{\text{—}}$$

where the state vector X belongs to a finite dimensional euclidean space and —~— is a closed and bounded set called the control constraint set in the S dimensional space of the control variable. Any $m(t) \in$ —~— is called an admissible control. A general optimal control problem requires the evaluation of $m(t) \in$ —~— which moves the dynamic system from the given initial state X_o to the desired state X^D so as to optimise the expected value of a good performance criterion $I_N = f^*\left[X(t), m(t)\right]$ The problem of optimal control can be viewed as a multistage decision process and naturally the optimum control law can be computed by using Dynamic programming techniques.

7.1 Accessible State Variables:-

Here, we assume that all state variables X are accessible for measurement and observation. Our problem is to determine the control law $\{ m(i) \}$; i = 1,2,...(n-1) which will move the system from any arbitrary initial state X(o) to any desired state X(t) so that the expected value of a good performance criterion I_N is optimised without violating the constraints on the process.

$X(k+1) = F(k)\ X\ (K) + G(K)m(K) + u(K)$

and the performance index (P.I.) chosen is

$$I_N = \sum_{K=1}^{N} \left[X^1(K)Q(K)\ X\ (K) + m^1(K-1)\ H(K-1)\ m(K-1)\right]$$

where Q and H are + ve definite symmetric matrices and this guarantees uniquearess and linearity of the control system. I_N implicitly contains the system model since it gives an account of the deviation of the system at any time from the desired state.

The elements of Q can be selected to give appropriate weighting to the response errors. The Lagrengian multiplier λ^* can be adjusted to avoid saturating controls while the elements of H can be chosen to take care of the cost of control. Undesirable control signals can be suppressed by giving very high values to the corresponding elements of H.

In an educational system the elements of Q can be selected to give appropriate penalty for not meeting the demands by the manpower and economic systems. Also, this can be used to give proper weighting for the social and cultural fulfilment of people if they are included as state variables.

The elements of H can be adjusted to avoid certain control signals which can not be applied in a practical system. If, some of the control signals (inputs) are too large and exceed the capacity of an educational activity (saturated), then λ^* can be adjusted to avoid such situations.

Considering the problem as a (N-j) stage decision process, the optimimum value of m(j) which will minimise the performance index I_N can be shown to be

$$m^o(j) = B(N-j)\, X(j) \qquad (17\,a)$$

$$\text{where } B(N-j) = -\left[L_{22}^{(N-\overline{j+1})} + \lambda^* H(j)\right]^{-1} L_{21}^{(N-j+1)} \qquad (17)$$

$$L_{21}^{(N-\overline{j+1})} = G^1(j)\, S^{(N-\overline{j+1})}\, F(j)$$

$$L_{22}^{(N-\overline{j+1})} = Q(j+1) + \hat{P}^{(N-\overline{j+1})}$$

$$S^{(N-\overline{j+1})} = Q(j+1) + P^{(N-\overline{j+1})}$$

$$P(N-j) = \left[L_{11}^{(N-\overline{j+1})} + L_{12}^{(N-\overline{j+1})}\, B(N-j)\right] \qquad (18)$$

$$L_{11}^{(N-\overline{j+1})} = F_1^1(j)\, S^{(N-\overline{j+1})}\, F(j)$$

$$= F^1(j)\, S^{(N-\overline{j+1})}\, G(j)$$

$$L_{12}^{(N-\overline{j+1})} = \hat{P}^{12} = o \text{ where } j = n-1$$

From (17) and (18)

$$B(1) = -\left[L_{22}(o) + L^{*}H\,(n-1)\right]^{-1} L_{21}(o)$$

$$\hat{P}\,(1) = \left[L_{11}(o) + L_{12}(o)B(1)\right]$$

when $j = n-2$, we get $B(2)$ which can be used to compute $\hat{P}(2)$ and thus (17) and (18) can be used to compute $m^{o}(j)$ recursively.

It is important to note that the feedback matrix B is time varying even if the system itself is time invariant and the optimal control policy vector is a function of all state variables. The control system is optimum even if the process is subjected to additive independent random noise in the sense that the expected value of I_N is minimised.

When the number of control signals is equal to the number of state variables and control signals are unbounded ($\lambda^{*} = o$)

$$B(n-j) = -\left[G^{1}(j)\,Q(j+1)\,G(j)\right]^{-1} G^{1}(j)\,Q(j+1)\,F(j)$$

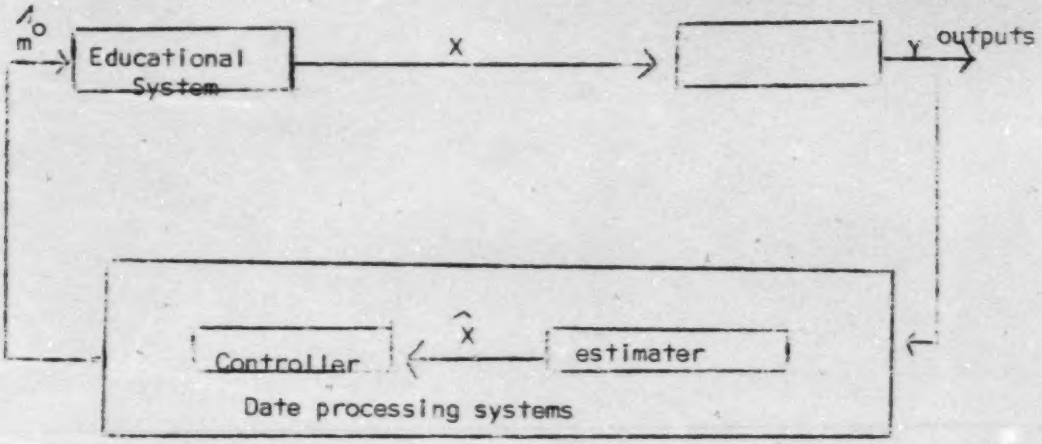
when the process is timeinvarient and the P.I. is not time weighted the feedback matrix B is timeinvariant.

$$B = -\left[G^{1}(T)QG(T)\right]^{-1} G^{1}(T)QF(T)$$

and optimum control can be acheived by feeding back all the state variables with appropriate weighting factors during each sampling period.

7.2 OPTIMAL CONTROL - INACCESSIBLE STATE VARIABLE

The state-space representation of a system is not unique and the importance of selecting measurable quantities (outputs) as state variables has already been mentioned. However, in many physical systems, some of the state variables are inaccessible for direct measurement and observation and furthermore those that are measured are often contaminated with noise. Attainments, ability, effect of social and family background etc of students can not be measured accurately. Optimal decision making systems are generally composed of an optimimum estimater and optimum controller.



It can be shown that

$$m^{o}(k/k) = B(N-k)\,\hat{X}(k/k) \qquad 15a$$

When the estimation error is a minimum, where

$\hat{m}^{o}$ = optimum estimate of m^{o}

and $\hat{x}$ = optimum estimate of x based upon $Y(k), Y(k-1), \ldots Y_{(o)}$

8 PLANNING MODEL:-

Forecasting models do tell us what might happen if certain conditions exist. But it is rather impossible to predict accurately these conditions which will determine the destiny of the educational systems. Moreover, forecasting models do not suggest what logical action should be taken at the right time to change the course of the educational system. We can not rely upon common sense methods while dealing with large amounts of data which requires enormous amount of processing time and skill. Obviously, the educational system will drift away from the desired operating point if the system is not allowed to adjust itself on the basis of its own outputs:

Planning models or feedback control systems modify their inputs in a specified way by observing their own outputs. The theory of optimal control discussed earlier can be used to find the input vector (control vector) m in equation (8) in an optimum way. Of course the performance index chosen for the analysis decides in what way the system is optimum.

We do not claim that a quadratic P.I. is sufficient to define the objectives of an educational system. This is only a preliminary approach.

The model (8) is a timeinvariant deterministic process. But, the theory of optimal control is applicable even if the educational system is a random parameter system.

The present deterministic systems approach is only a stepping stone to the future theory of control of actual educational systems.

The value of m in (8) can be found in an optimum way from the relation (17a). In our example in section 5, the elements of m are the new entrants into the system at the beginning of each academic year. If X^D is the desired output vector at the target year and X(o) is the vector at the base year then, m can be found for each year so that I_N is minimised.

If the computed value of m and the normal new entrants (e.g. entrants to primary can not be restricted) vary too much, the Langrangion multiplier λ^* and the weighting factors of H and Q can be adjusted to obtain desired accuracy.

Positive and negative values of m can be interpreted according to the state of the system. A value (section 6) of $m_2(t)$ indicates that m_2 new entrants are required at time t into level 2 to achieve the target figures. If this can not be supplied from outside wources we have many alternatives, such as

(1) increase the numbers promoted to level 2 at time t and this means better teaching, power etc. at time (t-1) in level 1,

(2) decrease promotions at time t from level 2 etc.

The value of m(KT) can be computed for all the years in the plan period (base year to target year) and these results will throw some light on the course of the educational system and policy makers can study the

effect of various decisions by simulation studies and take the necessary steps at the right time.

9 CONCLUSIONS

A unified systems theory approach in educational planning is bound to give deeper insight into the behaviour of the educational system.

The theory of optimal control as applied to the educational system can be really appreciated only if we can build a mathematical model which incorporates all the significant modes of system behaviour. In our discussion, we have considered the new entrants as the input vector m and the stock of students as the state vector X. In fact, both of these vectors should be expanded to include all the state and control variables that affect the performance of the educational systems. This leads to the conclusion that we should know more about the dynamics of the educational system (which deals with human beings) before we can apply with confidence the theory of optimal control in the design and analysis of educational systems. However, there is ample scope for experimentation with the approximate model developed here and the results should be useful.

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